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How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objectives tie to your state **Standards**

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**

Note exemplar pacing in the **Lesson Agenda**

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models

Standard(s): **3.4K** solve one-step and two-step problems involving multiplication and division within 100 using strategies based on objects, pictorial models, including arrays, area models, and equal groups; properties of operations; or recall of facts

Necessary Materials and Pre-Lesson Prep

- (S) Multiply by 2 (1–5) Pattern Sheet
- (S) Threes array no fill template
- (S) Personal white board
- (S) Blank paper

Lesson Agenda	Time
I. Do Now (source: fluency #1)	5 min
II. Fluency*	8 min
III. Concept Development	25 min
IV. Student Practice	15 min
V. Student Debrief	7 min
VI. Exit Ticket*	5 min

Mathematical Goal of this Lesson
Students learn they can use decomposition to break one larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

Opportunities to CFU

- ✓ Concept Development, by way of eliciting student responses
- ✓ Problems Set problems: #2, #3

Important Vocabulary

- array
- bracket**
- columns
- rows
- unit(s)

In this lesson, students are NOT responsible for the vocabulary distributive property. Please withhold as it will come up in later lessons.

Other Notes to Inform Your Planning

For Do Now: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.

For Fluency: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.

For Concept Development: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.

For Student Practice: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.

For Student Debrief: consider using the Eureka assigned Exit Ticket for whole group debrief exercise; Suggested strategy – guided discourse.

For Exit Ticket: Use Homework problems 2 & 3 for this lesson's Exit Ticket.

Other Notes: Though not formally discussed yet, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

Lesson Look Fors

Look for teachers to...

- Have established a signaling routine for choral response or work show during the respective fluency activities
- Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array
- Make the focus of the lesson understanding the visual representations

Look for students to...

- Explain what they see in the array and how it relates to a given number sentence.

Student Criteria for Success

- Shading, brackets, and/or dotted lines on an array will have mathematical significance
- brackets can identify parts or wholes
- dotted lines and shading represent decompositions
- We count units; in an array, counting rows is the same as counting units.
- Addition/subtraction and multiplication math facts (up to 4)
- Interpret an array
- Identify decompositions within an array
- Relate an annotated or labeled array to one or more number sentences
- Addition/subtraction (+/- up to 4)
- Multiplication (2, 3, and 4)

UNIT SYNOPSIS

In Unit 4, we explored the basic definitions, properties, graphs, and applications of trigonometric and inverse trigonometric functions. In the first half of this unit, we will apply those properties and use them to derive fundamental identities to evaluate trigonometric functions, simplify trigonometric expressions, develop additional trigonometric identities, and solve trigonometric equations.

The last half of this unit, we will explore techniques for solving oblique triangles using the Law of Sines and Law of Cosines. It is imperative that these techniques are explored conceptually so that students can apply the appropriate law in a given setting (e.g., solving AAS and SSS triangles will not require the same law).

CONTENT STANDARDS

Below are the standards addressed in this unit.

Texas Essential Knowledge and Skills (TEKS)	
Knowledge and Skills	Student Expectations (SE)
<p>(4) Number and Measure The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems.</p>	<p>(4.F) Use trigonometry in mathematical and real-world problems, including directional bearing. (4.G) Use the Law of Sines in mathematical and real-world problems. (4.H) Use the Law of Cosines in mathematical and real-world problems.</p>
<p>(5) Algebraic Reasoning The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms.</p>	<p>(5.M) Use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions. (5.N) Generate and solve trigonometric equations in mathematical and real-world problems.</p>

<p>Focus on Disciplinary Literacy</p> 	<p>Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.</p>
	<p>Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.</p>

LEARNING SUPPORTS BY LESSON

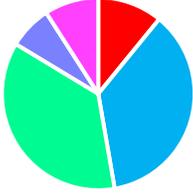
There is a checkmark for the math support if the lesson	Lessons →	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12
	Math Supports												
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓			✓				✓	✓	✓		✓
makes use of graphic organizers	Graphic Organizers	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives										✓		
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓		✓	✓	✓	✓		✓	✓		✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓			✓		✓	✓	
includes strategies that support language development													
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓					✓		✓		✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share			✓	✓	✓		✓		✓		✓	
includes sentence stems to support students with explanations	- Sentence Stems												
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics												
includes websites or equipment that enhances the lesson	Technological Support										✓	✓	✓
content can be presented in different forms													
uses hands-on tools or manipulatives to represent the math	- Concrete												
uses drawings to represent the math	- Pictorial	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

ROADMAP

AT A GLANCE: Unit 5 - Analytic Trigonometry			
Day	Date	Lesson	Lesson Title
1		1	Introduction to Trigonometric Identities
2		2	Simplifying Trigonometric Expressions
3			<i>Unit 5 Success Day Alpha – Review Trig Identities & Simplifying Trig Expressions</i>
4		3	Proving Trigonometric Equations (Basic)
5		4	Proving Trigonometric Identities (Advanced)
6		5	Solving Trigonometric Equations
7			<i>Unit 5 Success Day Beta – Review Trig Equations</i>
8		6	Sum and Difference Trig Identities
9		7	Double and Half Angle Trig Identities
10		8	Solving Trigonometric Equations (Advanced)
11			<i>Unit 5 Success Day Gamma – Review Advanced Trig Equations</i>
12		9	Law of Sines to Solve Triangles (AAS, ASA)
13		10	Law of Sines: The Ambiguous Case (SSA)
14		11	Law of Cosines to Solve Triangles (SAS, SSS)
15			<i>Unit 5 Success Day Delta – Review Law of Sines and Cosines</i>
16		12	Law of Sines and Cosines Application Problems
17			<i>Unit 5 Success Day Epsilon – Review topics based on your data</i>
18			<i>Unit 5 Success Day Zeta – Unit Assessment Review</i>
19			End of Unit 5 Assessment

Date: _____												
Lesson 1: Introduction to Trigonometric Identities												
<p>Standard(s)</p> <p>◆ (5.M) Use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions.</p>	<p>Notes for Intellectual Preparation & Lesson Planning</p> <p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ Graphing calculators ▪ Student Laptops for Desmos ▪ Document camera <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Lesson Structure:</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20px; background-color: red; border: 1px solid black;"></td> <td>Do Now (6 min)</td> </tr> <tr> <td style="width: 20px; background-color: blue; border: 1px solid black;"></td> <td>INM (30 min)</td> </tr> <tr> <td style="width: 20px; background-color: green; border: 1px solid black;"></td> <td>Student Practice (10 min)</td> </tr> <tr> <td style="width: 20px; background-color: purple; border: 1px solid black;"></td> <td>Debrief (4 min)</td> </tr> <tr> <td style="width: 20px; background-color: pink; border: 1px solid black;"></td> <td>Exit Ticket (5 min)</td> </tr> </table>  </div> <p>Mathematical Goal of this Lesson</p> <p>The lesson is mostly teacher-driven, with time built in for students to check the identities using Desmos.com. Once the students have a solid understanding of the derivation of all the new identities, they will apply them to evaluate, define, and manipulate given trigonometric problems. By the end of the lesson, students will state and apply the reciprocal, quotient, and Pythagorean identities.</p> <p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ What is the difference between an identity and an equation? ✓ How can we use quotient identities to derive the remaining Pythagorean identities? <div style="border: 1px solid black; padding: 5px; margin: 10px 0; text-align: center;"> <p>Focus on Disciplinary Literacy</p>  <p>INM</p> </div> <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> • This lesson introduces the basic trigonometric identities that can be derived from some of the concepts in Unit 4. In Lessons 2 through 8, students will be working with trigonometric identities to derive newer identities and apply those to solve trigonometric equations over the real number line or a subset of \mathbb{R}. • In Unit 6, students will work with implicit relations involving trigonometric functions defined by $x(t)$ and $y(t)$, parametric equations. 		Do Now (6 min)		INM (30 min)		Student Practice (10 min)		Debrief (4 min)		Exit Ticket (5 min)	<p>Lesson Look Fors</p> <p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Connect the Do Now with the concept of an identity in the INM. <input type="checkbox"/> Use Desmos to model $\sin x$ and $\cos\left(\frac{\pi}{2} - x\right)$ to investigate their relationships. <input type="checkbox"/> Manipulate the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ to derive the other two. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Define and differentiate between identities and equations. <input type="checkbox"/> Recall basic trig identities from Unit 4. <input type="checkbox"/> Discuss the relationship between sine and cosine when exploring the cofunction identities.
	Do Now (6 min)											
	INM (30 min)											
	Student Practice (10 min)											
	Debrief (4 min)											
	Exit Ticket (5 min)											
<p>Important Vocabulary</p> <ul style="list-style-type: none"> ▪ Trigonometric Identities ▪ Reciprocal Identities ▪ Quotient Identities ▪ Pythagorean Identities ▪ Cofunction Identities ▪ Odd-Even Identities 	<p>Student Know/Do Chart</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%; text-align: center; vertical-align: top;"></td> <td>An identity is an equation that is true for all values of the variable(s) for which the equation is defined.</td> </tr> <tr> <td style="text-align: center; vertical-align: top;"></td> <td>Some trigonometric identities follow directly from the definitions of the six basic trigonometric functions.</td> </tr> <tr> <td style="text-align: center; vertical-align: top;"></td> <td>Define and apply trigonometric identities.</td> </tr> <tr> <td style="text-align: center; vertical-align: top;"></td> <td>Use basic identities to define newer identities.</td> </tr> </table>			An identity is an equation that is true for all values of the variable(s) for which the equation is defined.		Some trigonometric identities follow directly from the definitions of the six basic trigonometric functions.		Define and apply trigonometric identities.		Use basic identities to define newer identities.		
	An identity is an equation that is true for all values of the variable(s) for which the equation is defined.											
	Some trigonometric identities follow directly from the definitions of the six basic trigonometric functions.											
	Define and apply trigonometric identities.											
	Use basic identities to define newer identities.											

Date: _____		
Lesson 2: Simplifying Trigonometric Expressions		
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
<p>◆ (5.M) Use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions.</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ Graphing calculators ▪ Document camera <div style="border: 1px solid #0070c0; padding: 5px; margin: 10px 0;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> ■ Do Now (7 min) ■ INM (15 min) ■ Student Practice (27 min) ■ Debrief (3 min) ■ Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson</p> <p>This lesson relies heavily on students fully understanding the identities derived in the previous lesson. Throughout the lesson, students will be asked to simplify, or reduce, trigonometric expressions using the trigonometric identities from Lesson 1.</p> <p>Students will learn five strategies for approaching simplification: Rewriting using sine and cosine, applying Pythagorean identities, splitting fractions, finding common denominators, and factoring. These are not all the possible strategies but will get the students to learn some main manipulation techniques.</p>	<p>Look for teachers to...</p> <ul style="list-style-type: none"> <input type="checkbox"/> Activates students' prior knowledge of basic trig identities to simply trigonometric expressions. <input type="checkbox"/> Modeling examples in the INM and probing questions stated in the TE script. <p>Look for students to...</p> <ul style="list-style-type: none"> <input type="checkbox"/> Use the basic trig identities to simplify trigonometric expressions. <input type="checkbox"/> Identify which trig identities can be applied for a given example.
Important Vocabulary	Student Know/Do Chart	
<ul style="list-style-type: none"> ▪ Trigonometric Expressions ▪ Simplify 	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ How can identities help us simplify trig expressions? ✓ When you're stuck in simplifying, how can you represent terms involving $\tan \theta$ and $\cot \theta$? <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> • This lesson extends on the basic trigonometric identities that can be derived from some of the concepts in Unit 4. In Lessons 3 through 8, students will be working with trigonometric identities to derive newer identities and apply those to solve trigonometric equations over the real number line or a subset of \mathbb{R}. • In Unit 6, students will work with implicit relations involving trigonometric functions defined by $x(t)$ and $y(t)$, parametric equations. 	<p>Know Some trigonometric identities follow directly from the definitions of the six basic trigonometric functions.</p> <p>Know Identities along with algebraic techniques (e.g., factoring or combining fractions over a common denominator) are used to simplify the expressions before dealing with them.</p> <p>Do Simplify trigonometric expressions by applying trigonometric identities.</p>

Date: _____		
Lesson 3: Proving Trigonometric Equations (Basic)		
<p>Standard(s)</p> <p>◆ (5.M) Use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions.</p>	<p>Notes for Intellectual Preparation & Lesson Planning</p> <p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ Graphing calculators ▪ Document camera <div data-bbox="485 305 1215 574" style="border: 1px solid black; padding: 5px;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> ■ Do Now (6 min) ■ INM (20 min) ■ Student Practice (20 min) ■ Debrief (4 min) ■ Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson</p> <p>Today's lesson is a direct continuation of the previous lesson: from simplifying expressions to verifying equations. In each problem today, students will be given an equation with trigonometric functions on either or both sides. The main goal behind verifying equations is to use logical steps to show that one side of the equation can be transformed into the other side of the equation. This manipulation can be done using the trigonometric identities and the strategies learned in Lesson 2.</p>	<p>Lesson Look Fors</p> <p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Activate students' prior knowledge of basic trig identities to prove trigonometric equations. <input type="checkbox"/> Incorporate CFUs over what strategies or entry points the students can choose to begin proving equations. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Choose the more "complicated" side of the equation to begin proving the equation.
<p>Important Vocabulary</p>	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ What identity can you use to prove trig equations involving a squared term? ✓ What is a good first step when you're unsure how to start proving a trig equation? 	<p>Student Know/Do Chart</p>
<ul style="list-style-type: none"> ▪ Verify Trigonometric Identity ▪ Prove Trigonometric Identity 	<div data-bbox="632 938 953 1105" style="border: 1px solid black; padding: 5px; text-align: center;"> <p>Focus on Disciplinary Literacy</p>  <p>Student Practice #1</p> </div> <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> • This lesson extends on the basic trigonometric identities that can be derived from some of the concepts in Unit 4. In Lessons 4 through 8, students will be working with trigonometric identities to derive newer identities and apply those to solve trigonometric equations over the real number line or a subset of \mathbb{R}. • In Unit 6, students will work with implicit relations involving trigonometric functions defined by $x(t)$ and $y(t)$, parametric equations. 	<ul style="list-style-type: none">  Some trigonometric identities follow directly from the definitions of the six basic trigonometric functions.  Identities along with algebraic techniques (e.g., factoring or combining fractions over a common denominator) are used to simplify the expressions before dealing with proving.  Use trigonometric identities to rewrite or simply trig expression.  Prove trig equations.

Date: _____		
Lesson 4: Proving Trigonometric Identities (Advanced)		
<p>Standard(s)</p> <p>◆ (5.M) Use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions.</p>	<p>Notes for Intellectual Preparation & Lesson Planning</p> <p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> Graphing calculators Document camera <div data-bbox="485 302 1220 573" style="border: 1px solid black; padding: 5px;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> ■ Do Now (7 min) ■ INM (20 min) ■ Student Practice (20 min) ■ Debrief (3 min) ■ Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson</p> <p>Today's lesson is a deeper dive into verifying trigonometric equations. The goal, like yesterday, is to manipulate one side of the equation to match the other side. This manipulation can involve the use of trigonometric identities, or strategies, like factoring or finding the common denominator.</p> <p>Today, we will be investigating an additional strategy: multiplying by the conjugate. The conjugate of a binomial is found by changing the negative/positive sign between the two terms. In Algebra 2, students learned that multiplying two conjugates will result in a difference of squares. Investigating this concept can also help students readily identify how to factor difference of squares.</p>	<p>Lesson Look Fors</p> <p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Activates students' prior knowledge of basic trig identities to prove trigonometric equations. <input type="checkbox"/> Incorporates CFUs over what strategies or entry points the students can choose to begin proving equations. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Choose the more "complicated" side of the equation to begin proving the equation. <input type="checkbox"/> Explain what a conjugate pair is.
<p>Important Vocabulary</p> <ul style="list-style-type: none"> Verify Trigonometric Identity Prove Trigonometric Identity 	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ What is the conjugate of $a + b$? ✓ How do conjugates of trig expressions help us simply and prove trig equations? <div data-bbox="1194 1075 1478 1239" style="border: 1px solid black; padding: 5px; text-align: center;"> <p>Focus on Disciplinary Literacy</p>  <p>INM</p> </div> <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> This lesson follows from the previous. See Lesson 3's Other Notes to Inform Planning. 	<p>Student Know/Do Chart</p> <p>Know  Some trigonometric identities follow directly from the definitions of the six basic trigonometric functions.</p> <p>Know  Identities along with algebraic techniques (e.g., factoring or combining fractions over a common denominator) are used to simplify the expressions before dealing with proving.</p> <p>Do  Use trigonometric identities to rewrite or simply trig expression.</p> <p>Do  Prove trig equations.</p>

Date: _____												
Lesson 5: Solving Trigonometric Equations												
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors										
<p>◆ (5.N) Generate and solve trigonometric equations in mathematical and real-world problems.</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> Graphing calculators Document camera <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Lesson Structure:</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20px; background-color: red; border: 1px solid black;"></td> <td>Do Now (8 min)</td> </tr> <tr> <td style="width: 20px; background-color: blue; border: 1px solid black;"></td> <td>INM (20 min)</td> </tr> <tr> <td style="width: 20px; background-color: green; border: 1px solid black;"></td> <td>Student Practice (18 min)</td> </tr> <tr> <td style="width: 20px; background-color: purple; border: 1px solid black;"></td> <td>Debrief (4 min)</td> </tr> <tr> <td style="width: 20px; background-color: pink; border: 1px solid black;"></td> <td>Exit Ticket (5 min)</td> </tr> </table>  </div> <p>Mathematical Goal of this Lesson In today's lesson, students will need to recall several properties of the Unit Circle, particularly the value and positivity and negativity of each trigonometric function at the key reference angles, as well as the periods of the trigonometric functions.</p> <p>Students will be asked to solve trigonometric equations, which means they will need to find the value, or values that make the equation true. To do this, students will use similar techniques to solving other algebraic equations to isolate the trigonometric function, then use their knowledge of the Unit Circle to find the correct value.</p>		Do Now (8 min)		INM (20 min)		Student Practice (18 min)		Debrief (4 min)		Exit Ticket (5 min)	<p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> Activate students' prior knowledge of basic trig identities to rewrite trig expressions. Incorporate the unit circle to explain how solutions to a trig equation may be found and how many. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> Apply trig identities to simplify and set up trig equations. Use the unit circle to verify or assist in solving trig equations.
		Do Now (8 min)										
	INM (20 min)											
	Student Practice (18 min)											
	Debrief (4 min)											
	Exit Ticket (5 min)											
<p>Important Vocabulary</p> <ul style="list-style-type: none"> Trigonometric Equations "Find All Solutions" 	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> How many solutions do trigonometric equations have over \mathbb{R}? How can the unit circle assist in finding <i>all</i> solutions to trigonometric equation? <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> Students will solve trigonometric equations in abstract and contextual settings. During Lessons 9 through 12, students will solve trig equations using the Law of Sines/Cosines. In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam. In this problem, students need to derive $x(t)$ and find the times t for which $v(t) = 0$, then calculate acceleration. <hr/> <p>28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?</p> <p>(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$</p>	<p>Student Know/Do Chart</p> <p>Know Since trigonometric functions are periodic, the output values repeat. To "find all solutions" means to account for the domain beyond the Unit Circle.</p> <p>Know The period of a trigonometric function illustrates the time between repeated values.</p> <p>Do Solve trigonometric equations.</p>										

Date: _____										
Lesson 6: Sum and Difference Trig Identities										
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors								
<p>◆ (5.M) Use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions.</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> Graphing calculators Document camera 	<p>Look for teachers to...</p> <ul style="list-style-type: none"> Activate students' prior knowledge of basic trig identities to rewrite trig expressions. Incorporate the unit circle to explain how solutions to a trig equation may be found and how many. <p>Look for students to...</p> <ul style="list-style-type: none"> Apply trig identities to simplify and set up trig equations. Use the unit circle to verify or assist in solving trig equations. 								
	<div style="border: 1px solid black; padding: 5px;"> <p>Lesson Structure:</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20px; background-color: red; border: 1px solid black;"></td> <td>Do Now (6 min)</td> </tr> <tr> <td style="width: 20px; background-color: blue; border: 1px solid black;"></td> <td>INM (20 min)</td> </tr> <tr> <td style="width: 20px; background-color: green; border: 1px solid black;"></td> <td>Student Practice (18 min)</td> </tr> <tr> <td style="width: 20px; background-color: purple; border: 1px solid black;"></td> <td>Debrief (3 min)</td> </tr> <tr> <td style="width: 20px; background-color: pink; border: 1px solid black;"></td> <td>Exit Ticket (5 min)</td> </tr> </table>  </div> <p>Mathematical Goal of this Lesson</p> <p>In this lesson, students will be expanding their trigonometric identity toolbox with the sum and difference identities. There is a sum and difference identity for sine, cosine, and tangent; however, the proofs are time-consuming. The sine and cosine identities have a very interesting geometric proof if you would like to give that as an extension for your stronger students.</p>			Do Now (6 min)		INM (20 min)		Student Practice (18 min)		Debrief (3 min)
	Do Now (6 min)									
	INM (20 min)									
	Student Practice (18 min)									
	Debrief (3 min)									
	Exit Ticket (5 min)									
<p>Important Vocabulary</p> <ul style="list-style-type: none"> Sum and Difference Identities 	<p>The bulk of the lesson will be spent learning how to apply these identities. Organization is highly important in this lesson, so drawing and labeling diagrams of the main angles will be the first step, followed by writing out the given sine, cosine, and tangent values. Students will practice with angles given in both radians and degrees. Additionally, they will investigate the differences between the identities, so they will be able to work backwards to find values as well.</p> <p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ What are some similarities and differences between the sum/diff trig identities for sine and cosine? ✓ How can you use the sum/diff trig identities in multiple ways? <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> Students will solve trigonometric equations in abstract and contextual settings. During Lessons 9 through 12, students will solve trig equations using the Law of Sines/Cosines. In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam. In this problem, students need to derive $x(t)$ and find the times t for which $v(t) = 0$, then calculate acceleration. <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p style="text-align: center;">Focus on Disciplinary Literacy</p>  <p style="text-align: center;">INM</p> </div>	<p>Student Know/Do Chart</p> <p>Know The trigonometric value of additional angle inputs can be calculated using the sum and difference of the main angles in standard position.</p> <p>Know Identities along with algebraic techniques (e.g., factoring or combining fractions over a common denominator) are used to simplify the expressions before dealing with solving trig equations.</p> <p>Do Rewrite trigonometric expressions using sum/diff rule.</p> <p>Do Solve trigonometric equations.</p>								
	<p>28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?</p> <p>(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$</p>									

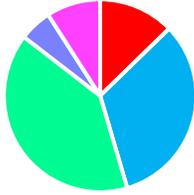
Date: _____		
Lesson 7: Double and Half Angle Trig Identities		
Standard(s) ◆ (5.M) Use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions.	Notes for Intellectual Preparation & Lesson Planning Necessary Materials and Pre-Lesson Prep <ul style="list-style-type: none"> Graphing calculators Document camera <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> Lesson Structure:  <ul style="list-style-type: none"> Do Now (6 min) INM (26 min) Student Practice (15 min) Debrief (3 min) Exit Ticket (5 min) </div> Mathematical Goal of this Lesson In this lesson, students will prove at least 6 new identities. Feel free to gauge your classroom and adjust accordingly if you feel like the proofs are too time-consuming; however, the sine and cosine double angle identities are a simpler proof and should be covered.	Lesson Look Fors Look for teachers to... <ul style="list-style-type: none"> Activate students' prior knowledge of basic trig identities to rewrite trig expressions. Incorporate the unit circle to explain how solutions to a trig equation may be found and how many. Look for students to... <ul style="list-style-type: none"> Apply trig identities to simplify and set up trig equations. Use the unit circle to verify or assist in solving trig equations.
Important Vocabulary	More time will be spent with teacher-led proofs today, but there is still time for students to practice application independently, as well as in groups.	Student Know/Do Chart
<ul style="list-style-type: none"> Double Angle Formulas Half Angle Formulas 	Opportunities to CFU <ul style="list-style-type: none"> How can you check if you have an erroneous solution? (INM) When dealing with multiple logs in an equation, what can we apply? (INM) How does the 1-1 property assist in solving logarithmic equations? (INM) Other Notes to Inform Your Planning <ul style="list-style-type: none"> Students will solve trigonometric equations in abstract and contextual settings. During Lessons 9 through 12, students will solve trig equations using the Law of Sines/Cosines. In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam. In this problem, students need to derive $x(t)$ and find the times t for which $v(t) = 0$, then calculate acceleration. <hr/> 28. For $t \geq 0$, the position of a particle moving along the x -axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ? (A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$	<div style="margin-bottom: 10px;">  The trigonometric value of additional angle inputs can be calculated using the double and half of the main angles in standard position. </div> <div style="margin-bottom: 10px;">  Identities along with algebraic techniques (e.g., factoring or combining fractions over a common denominator) are used to simplify the expressions before dealing with solving trig equations. </div> <div style="margin-bottom: 10px;">  Rewrite trigonometric expressions using double/half angle rule. </div> <div>  Solve trigonometric equations. </div>

Date: _____												
Lesson 8: Solving Trigonometric Equations (Advanced)												
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors										
<p>◆ (5.N) Generate and solve trigonometric equations in mathematical and real-world problems.</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> Graphing calculators Document camera <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Lesson Structure:</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20px; text-align: center;">■</td> <td>Do Now (6 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>INM (22 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Student Practice (19 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Debrief (3 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Exit Ticket (5 min)</td> </tr> </table>  </div> <p>Mathematical Goal of this Lesson Today's lesson is the second on solving trigonometric equations. After a Do Now of applying the double and half angle identities, students will work in pairs to review the previous strategies from Lesson 5. In addition to the previous strategies, students will be asked to use the double and half angle identities as a method to rewrite parts of the equations.</p>	■	Do Now (6 min)	■	INM (22 min)	■	Student Practice (19 min)	■	Debrief (3 min)	■	Exit Ticket (5 min)	<p>Look for teachers to...</p> <ul style="list-style-type: none"> Activate students' prior knowledge of basic trig identities to rewrite trig expressions. Incorporate the unit circle to explain how solutions to a trig equation may be found and how many. <p>Look for students to...</p> <ul style="list-style-type: none"> Apply trig identities to simplify and set up trig equations. Use the unit circle to verify or assist in solving trig equations.
	■	Do Now (6 min)										
■	INM (22 min)											
■	Student Practice (19 min)											
■	Debrief (3 min)											
■	Exit Ticket (5 min)											
<p>Important Vocabulary</p> <ul style="list-style-type: none"> Trigonometric Equations "Find All Solutions" 	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> What are some cues from a problem that will help you identify what strategy to use? List the strategies you know that simplifies trig equations before solving them. <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> Students will solve trigonometric equations in abstract and contextual settings. During Lessons 9 through 12, students will solve trig equations using the Law of Sines/Cosines. In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam. In this problem, students need to derive $x(t)$ and find the times t for which $v(t) = 0$, then calculate acceleration. <hr/> <p>28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0?</p> <p>(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$</p>	<p>Student Know/Do Chart</p> <p> Since trigonometric functions are periodic, the output values repeat. To "find all solutions" means to account for the domain beyond the Unit Circle.</p> <p> The period of a trigonometric function illustrates the time between repeated values.</p> <p> Solve trigonometric equations.</p>										

Date: _____		
Lesson 9: Law of Sines to Solve Triangles (AAS, ASA)		
Standard(s) ◆ (4.G) Use the Law of Sines in mathematical and real-world problems.	Notes for Intellectual Preparation & Lesson Planning <u>Necessary Materials and Pre-Lesson Prep</u> <ul style="list-style-type: none"> ▪ Graphing calculators ▪ Document camera ▪ Bingo Boards ▪ Bingo Chips ▪ PowerPoint 5.9 <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> Lesson Structure: <ul style="list-style-type: none"> ■ Do Now (5 min) ■ INM (17 min) ■ Student Practice (25 min) ■ Debrief (3 min) ■ Exit Ticket (5 min)  </div> <u>Mathematical Goal of this Lesson</u> Today's lesson steps back from trigonometric functions within the Unit Circle, and dives into how they are used in Geometry. Students will start by discussing theorems and strategies that know to use with right triangles, mainly Pythagorean Theorem and basic trigonometry (SOH CAH TOA). However, in the real world, triangles are not required to have right angles, so students will be learning how to calculate the sides and angles of acute triangles.	Lesson Look Fors <u>Look for teachers to...</u> <ul style="list-style-type: none"> <input type="checkbox"/> Use diagrams or models to verify the legitimacy of a triangle given a set of parameters. <input type="checkbox"/> Use the bingo activity during student practice. <u>Look for students to...</u> <ul style="list-style-type: none"> <input type="checkbox"/> Identify the type of triangle given (ASA or AAS). <input type="checkbox"/> Apply the Law of Sines to solve the triangle.
Important Vocabulary <ul style="list-style-type: none"> ▪ Law of Sines ▪ Oblique Triangle ▪ Acute Triangle ▪ "Solve The Triangle" ▪ AAS ▪ ASA 	The bulk of the lesson will be spent on a Bingo game, which allows students to apply the Law of Sines to various triangles that they must draw. <u>Opportunities to CFU</u> <ul style="list-style-type: none"> ✓ Given a set of parameters of a triangle, how can you determine if it is legitimate (it can be solved)? ✓ Given a set of parameters, how do you know that you can use the Law of Sines? <u>Other Notes to Inform Your Planning</u> <ul style="list-style-type: none"> • The Law of Sines can only be used to solve triangle that are classified as AAS, ASA, and SSA. However, SSA triangles may have more than one set of solutions. This depends on the parameters of the triangle given. It may be that there is no solution, one or two possible triangles (two sets of solutions). This is called the Ambiguous Case. • Lesson 10 discuss the Ambiguous Case (SSA). <div style="border: 1px solid black; padding: 5px; margin: 10px 0; text-align: center;"> Focus on Disciplinary Literacy  INM </div>	Student Know/Do Chart <ul style="list-style-type: none">  These threesomes that determine triangle congruence are known by their acronyms: AAS, ASA, SAS, and SSS. The other two acronyms represent matchups that don't quite work: AAA determines similarity only, while SSA does not even determine similarity.  Law of Sines can be applied to triangle with given ASA or AAS.  Use the Law of Sines to solve AAS and ASA triangles.

Date: _____		
Lesson 10: Law of Sines: The Ambiguous Case (SSA)		
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
<p>◆ (4.G) Use the Law of Sines in mathematical and real-world problems.</p>	<p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ Graphing calculators ▪ Rulers ▪ Document camera ▪ Protractors ▪ GeoGebra Activity 	<p><u>Look for teachers to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Use diagrams or models to verify the legitimacy of a triangle given a set of parameters. <input type="checkbox"/> Use the GeoGebra Activity to model how SSA triangles may have no solution or up to two sets of solutions. <p><u>Look for students to...</u></p> <ul style="list-style-type: none"> <input type="checkbox"/> Identify the type of triangle given (ASA, AAS, or SSA). <input type="checkbox"/> Apply the Law of Sines to solve the triangle.
	<div style="border: 1px solid black; padding: 5px;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> ■ Do Now (5 min) ■ INM (25 min) ■ Student Practice (17 min) ■ Debrief (3 min) ■ Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson</p> <p>Today's lesson is a further examination into the Law of Sines. Students have already seen that they can apply the Law of Sines when they have been given AAS or ASA, because both situations provide a set of opposite angle and side to create one side of the proportion. The third set of givens used to apply Law of Sines is SSA, but it is not as straight forward, so it is called the Ambiguous Case. By the end of the lesson, students will solve SSA triangles in abstract and contextual settings.</p>	
Important Vocabulary	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ What are the conditions that would make it possible for zero, one, or two triangles to be created? ✓ What givens create an ambiguous case for Law of Sines? <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> • Recall from geometry that a triangle has six parts (three sides (S), three angles (A)), but that its size and shape can be completely determined by fixing only three of those parts, provided they are the right three. These threesomes that determine triangle congruence are known by their acronyms: AAS, ASA, SAS, and SSS. The other two acronyms represent matchups that don't quite work: AAA determines similarity only, while SSA does not even determine similarity. • The Law of Sines states that the ratio of the sine of an angle to the length of its opposite side is the same for all three angles of any triangle. 	<p>Student Know/Do Chart</p> <p> Law of Sines can be applied to triangle with given ASA or AAS.</p> <p> When a triangle has two sides and a non-included angle, this presents the ambiguous case which can create the opportunity for zero, one, or two triangles to be drawn.</p> <p> Use the Law of Sines to solve AAS, ASA, and SSA triangles.</p>
<ul style="list-style-type: none"> ▪ Law of Sines ▪ Oblique Triangle ▪ "Solve The Triangle" ▪ SSA 		

Date: _____		
Lesson 11: Law of Cosines to Solve Triangles (SAS, SSS)		
<p>Standard(s)</p> <ul style="list-style-type: none"> ◆ (4.H) Use the Law of Cosines in mathematical and real-world problems. ◆ (5.N) Generate and solve trigonometric equations in mathematical and real-world problems. 	<p>Notes for Intellectual Preparation & Lesson Planning</p> <p>Necessary Materials and Pre-Lesson Prep</p> <ul style="list-style-type: none"> ▪ Graphing calculators ▪ Document camera <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Lesson Structure:</p> <ul style="list-style-type: none"> ■ Do Now (5 min) ■ INM (22 min) ■ Student Practice (20 min) ■ Debrief (3 min) ■ Exit Ticket (5 min)  </div> <p>Mathematical Goal of this Lesson</p> <p>Today's lesson starts with the derivation of the Law of Cosines. Similar to the derivation of the Law of Sines, when presented with a non-right triangle, an altitude can be drawn splitting the angle into two right triangles. From there, right triangle trigonometry and Pythagorean Theorem can be applied to find the value of side lengths.</p> <p>After students work through an example with numbers, they will use a general triangle with no given angles or sides to derive the Law of Cosines.</p>	<p>Lesson Look Fors</p> <p>Look for teachers to...</p> <ul style="list-style-type: none"> <input type="checkbox"/> Facilitate a discussion for why Laws of Sines can't be applied for SSS and SAS triangles. <input type="checkbox"/> Engage students to verify their solutions by using tech or drawing their triangle. <p>Look for students to...</p> <ul style="list-style-type: none"> <input type="checkbox"/> Determine which law can and can't be applied. <input type="checkbox"/> Apply prior knowledge from Lessons 1 through 8 to solve triangles.
<p>Important Vocabulary</p> <ul style="list-style-type: none"> ▪ Law of Cosines ▪ Oblique Triangle ▪ "Solve The Triangle" ▪ SAS ▪ SSS 	<p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ What is the similarity between the Pythagorean theorem and Law of Cosines? ✓ How do you know you can apply either the Law of Cosine or Sine? <p>Other Notes to Inform Your Planning</p> <ul style="list-style-type: none"> • Recall from geometry that a triangle has six parts (three sides (S), three angles (A)), but that its size and shape can be completely determined by fixing only three of those parts, provided they are the right three. These threesomes that determine triangle congruence are known by their acronyms: AAS, ASA, SAS, and SSS. The other two acronyms represent matchups that don't quite work: AAA determines similarity only, while SSA does not even determine similarity. • Having seen the Law of Sines, you will probably not be surprised to learn that there is a Law of Cosines. There are many such parallels in mathematics. What you might find surprising is that the Law of Cosines has absolutely no resemblance to the Law of Sines. Instead, it resembles the Pythagorean Theorem. In fact, the Law of Cosines is often called the "generalized Pythagorean Theorem" because it contains that classic theorem as a special case. 	<p>Student Know/Do Chart</p> <ul style="list-style-type: none">  In order to apply the Law of Cosines, a triangle must have either two sides and an included angle (SAS), or all three sides (SSS).  Law of Cosine resemble the Pythagorean theorem.  Explain which is applicable to a given triangle.  Solve triangles using the Law of Cosines.

Date: _____												
Lesson 12: Law of Sines and Cosines Applications												
Standard(s) <ul style="list-style-type: none"> ◆ (4.G) Use the Law of Sines in mathematical and real-world problems. ◆ (4.H) Use the Law of Cosines in mathematical and real-world problems. ◆ (5.N) Generate and solve trigonometric equations in mathematical and real-world problems. 	Notes for Intellectual Preparation & Lesson Planning <u>Necessary Materials and Pre-Lesson Prep</u> <ul style="list-style-type: none"> ▪ Graphing calculators ▪ Laptops ▪ Document camera <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> Lesson Structure: <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20px; text-align: center;">■</td> <td>Do Now (7 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>INM (18 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Student Practice (22 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Debrief (3 min)</td> </tr> <tr> <td style="text-align: center;">■</td> <td>Exit Ticket (5 min)</td> </tr> </table>  </div> <p>Mathematical Goal of this Lesson Today's lesson is the concluding lesson of this unit and is a summation of applying Law of Sines and Law of Cosines in real world, or more complicated geometrical questions.</p> <p>Opportunities to CFU</p> <ul style="list-style-type: none"> ✓ How do you know you can apply either the Law of Cosine or Sine? ✓ How are theorems used with right triangles used with acute and oblique triangles? 	■	Do Now (7 min)	■	INM (18 min)	■	Student Practice (22 min)	■	Debrief (3 min)	■	Exit Ticket (5 min)	Lesson Look Fors <u>Look for teachers to...</u> <ul style="list-style-type: none"> <input type="checkbox"/> Connect prior knowledge over the laws of sines/cosines. <u>Look for students to...</u> <ul style="list-style-type: none"> <input type="checkbox"/> Determine which law can and can't be applied. <input type="checkbox"/> Apply prior knowledge from Lessons 1 through 8 to solve triangles.
■	Do Now (7 min)											
■	INM (18 min)											
■	Student Practice (22 min)											
■	Debrief (3 min)											
■	Exit Ticket (5 min)											
Important Vocabulary <ul style="list-style-type: none"> ▪ One-to-One ▪ arcsine ▪ arccosine ▪ arctangent 	<u>Other Notes to Inform Your Planning</u> <ul style="list-style-type: none"> • N/A <div style="border: 1px solid black; padding: 5px; margin-top: 10px; text-align: center;"> Focus on Disciplinary Literacy  INM </div>	Student Know/Do Chart <p>Know  In order to apply the Law of Cosines, a triangle must have either two sides and an included angle (SAS), or all three sides (SSS).</p> <p>Know  These threesomes that determine triangle congruence are known by their acronyms: AAS, ASA, SAS, and SSS. The other two acronyms represent matchups that don't quite work: AAA determines similarity only, while SSA does not even determine similarity.</p> <p>Do  Explain which is applicable to a given triangle.</p> <p>Do  Solve triangles using the Law of Cosines.</p>										

Recommended Unit 5 Success Day Material and Resources

Date: _____

To review **topics based on your data on Success Days**, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review or practice basic trig identities:

Content Video Lessons:

- Trigonometric Identities
- Basic Trig Identities
- Cofunction Identities
- Trigonometric Identities
- Simplifying Trigonometric Expressions, Part 1
- Simplifying Trigonometric Expressions, Part 2

To review or practice proving trig identities and solving trig equations:

Content Video Lessons:

- Verifying Trigonometric Identities
- Solving Trig Equations
- Solving Trig Equations on a Closed Interval

To review or practice sum and difference identities:

Content Video Lessons:

- Sum and Difference Formulas

Additional Resources:

- Angle addition identities

To review or practice double and half angle identities:

Content Video Lessons:

- Double Angle Formulas
- Using Double Angle Formulas

Remote Learning Resources:

- Using trigonometric Identities

To review or practice Law of Sines and Law of Cosines:

Additional Resources:

- Laws of sines
- Laws of cosines
- Solving general triangles

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification										
Standards	Specificity	Notes/Explanations/Examples								
<p>(5.N) Generate and solve trigonometric equations in mathematical and real-world problems.</p>	<p>Concepts: Trigonometric Identities:</p> <ul style="list-style-type: none"> • reciprocal identities <ul style="list-style-type: none"> ○ $\csc \theta = \frac{1}{\sin \theta}$ ○ $\sin \theta = \frac{1}{\csc \theta}$ ○ $\sec \theta = \frac{1}{\cos \theta}$ ○ $\cos \theta = \frac{1}{\sec \theta}$ ○ $\cot \theta = \frac{1}{\tan \theta}$ ○ $\tan \theta = \frac{1}{\cot \theta}$ • quotient identities <ul style="list-style-type: none"> ○ $\tan \theta = \frac{\sin \theta}{\cos \theta}$ ○ $\cot \theta = \frac{\cos \theta}{\sin \theta}$ • Pythagorean identities <ul style="list-style-type: none"> ○ $\cos^2 \theta + \sin^2 \theta = 1$ ○ $1 + \tan^2 \theta = \sec^2 \theta$ ○ $\cot^2 \theta + 1 = \csc^2 \theta$ • double angle formulas <ul style="list-style-type: none"> ○ $\sin 2x = 2 \sin x \cos x$ ○ $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$ ○ $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ 	<p>Geometry TEKS (6.B) prove two triangles are congruent by applying the Side-Angle-Side, Angle-Side-Angle, Side-Side-Side, Angle-Angle-Side, and Hypotenuse-Leg congruence conditions;</p> <p>6th Grade TEKS (8.A) extend previous knowledge of triangles and their properties to include the sum of angles of a triangle, the relationship between the lengths of sides and measures of angles in a triangle, and determining when three lengths form a triangle;</p> <p>AP Calculus AB 2016 Released FRQ #2</p> <p style="text-align: center;">AP[®] CALCULUS AB 2016 SCORING GUIDELINES</p> <p style="text-align: center;">Question 2</p> <p>For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$. The particle is at position $x = 2$ at time $t = 4$.</p> <p>(a) At time $t = 4$, is the particle speeding up or slowing down? (b) Find all times t in the interval $0 < t < 3$ when the particle changes direction. Justify your answer. (c) Find the position of the particle at time $t = 0$. (d) Find the total distance the particle travels from time $t = 0$ to time $t = 3$.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 60%; padding: 5px;"> <p>(a) $v(4) = 2.978716 > 0$ $v'(4) = -1.164000 < 0$</p> <p style="text-align: center;">The particle is slowing down since the velocity and acceleration have different signs.</p> </td> <td style="width: 40%; padding: 5px; vertical-align: top;"> <p>2 : conclusion with reason</p> </td> </tr> <tr> <td style="padding: 5px;"> <p>(b) $v(t) = 0 \Rightarrow t = 2.707468$</p> <p style="text-align: center;">$v(t)$ changes from positive to negative at $t = 2.707$. Therefore, the particle changes direction at this time.</p> </td> <td style="padding: 5px; vertical-align: top;"> <p>2 : $\left\{ \begin{array}{l} 1 : t = 2.707 \\ 1 : \text{justification} \end{array} \right.$</p> </td> </tr> <tr> <td style="padding: 5px;"> <p>(c) $x(0) = x(4) + \int_4^0 v(t) dt$ $= 2 + (-5.815027) = -3.815$</p> </td> <td style="padding: 5px; vertical-align: top;"> <p>3 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{array} \right.$</p> </td> </tr> <tr> <td style="padding: 5px;"> <p>(d) Distance = $\int_0^3 v(t) dt = 5.301$</p> </td> <td style="padding: 5px; vertical-align: top;"> <p>2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$</p> </td> </tr> </table>	<p>(a) $v(4) = 2.978716 > 0$ $v'(4) = -1.164000 < 0$</p> <p style="text-align: center;">The particle is slowing down since the velocity and acceleration have different signs.</p>	<p>2 : conclusion with reason</p>	<p>(b) $v(t) = 0 \Rightarrow t = 2.707468$</p> <p style="text-align: center;">$v(t)$ changes from positive to negative at $t = 2.707$. Therefore, the particle changes direction at this time.</p>	<p>2 : $\left\{ \begin{array}{l} 1 : t = 2.707 \\ 1 : \text{justification} \end{array} \right.$</p>	<p>(c) $x(0) = x(4) + \int_4^0 v(t) dt$ $= 2 + (-5.815027) = -3.815$</p>	<p>3 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{array} \right.$</p>	<p>(d) Distance = $\int_0^3 v(t) dt = 5.301$</p>	<p>2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$</p>
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VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students for** in the subsequent course.

Geometry	Pre-Calculus	AP Calculus AB/BC
<ul style="list-style-type: none"> • G.6D Verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, mid-segments, and medians, and apply these relationships to solve problems. • G.8A Prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems. • G.8B Identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems. • G.9A Determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems. • G.9B Apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean theorem, including Pythagorean triples, to solve problems. • G.12B Apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems. • G.12C Apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems. • G.12D Describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle. 	<ul style="list-style-type: none"> • (4.F) Use trigonometry in mathematical and real-world problems, including directional bearing. • (4.G) Use the Law of Sines in mathematical and real-world problems. • (4.H) Use the Law of Cosines in mathematical and real-world problems. • (5.M) Use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions. • (5.N) Generate and solve trigonometric equations in mathematical and real-world problems. 	<ul style="list-style-type: none"> • LIM-2.B.2 Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains. • FUN-3.A.4 Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions. • FUN-3.B.3 Rearranging tangent, cotangent, secant, and cosecant functions using identities allows differentiation using derivative rules. • FUN-3.C.1 The chain rule provides a way to differentiate composite functions. • FUN-3.E.1 The chain rule and definition of an inverse function can be used to find the derivative of an inverse function, provided the derivative exists. • FUN-3.E.2 The chain rule applied with the definition of an inverse function, or the formula for the derivative of an inverse function, can be used to find the derivative of inverse trigonometric functions. • FUN-4.A.1 The first derivative of a function can provide information about the function and its graph, including intervals where the function is increasing or decreasing. • FUN-4.A.2 The first derivative of a function can determine the location of relative (local) extrema of the function. • FUN-4.A.3 Absolute (global) extrema of a function on a closed interval can only occur at critical points or at endpoints. • FUN-4.A.4 The graph of a function is concave up (down) on an open interval if the function's derivative is increasing (decreasing) on that interval. • FUN-4.A.5 The second derivative of a function provides information about the function and its graph, including intervals of upward or downward concavity. • FUN-4.A.6 The second derivative of a function may be used to locate points of inflection for the graph of the original function. • FUN-4.A.7 The second derivative of a function may determine whether a critical point is the location of a relative (local) maximum or minimum. • CHA-3.D.1 The chain rule is the basis for differentiation variables in a related rates problem with respect to the same independent variable. • CHA-3.E.1 The derivative can be used to solve related rates problems; that is, finding a rate at which one quantity is changing by relating it to other quantities whose rate of change are known.
<h3>Algebra 2</h3>		
<ul style="list-style-type: none"> • 2A.2C Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. • 2A.2D Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other. 		<p>NOTE: These "essential knowledge" (EK) standards are from The College Board Concept Outline for AP Calculus AB/BC, not the TEKS.</p>